Finite Math - Spring 2019

Lecture Notes - 3/12/2019

Homework

- Section 1.2 - 5, 6, 7, 8, 29, 31, 33
- Graph the following lines:
  (1) $y = 2x$, (2) $2x + 3y = 0$, (3) $5x - 6y = 0$
- Section 4.1 - 1, 5, 7, 9, 10, 11, 12, 13, 15, 17, 20, 21, 23, 25, 26, 27, 28, 31, 33, 77, 78, 79, 80

Section 1.2 - Graphs and Lines

Definition 1 (Line)**.** *A* line *is the graph of an equation of the form*

$$Ax + By = C$$

where not both of A and B are equal to zero (i.e., if $A = 0$, then $B \neq 0$ and vice-versa).

Graphing Lines. There are two common ways of graphing lines: by **finding intercepts** and by **using the slope and a point**. We will focus on the method of finding intercepts here in the notes. You can read about using the slope to graph a line in the textbook.

Finding Intercepts.

Definition 2 (Intercept)**.** *A point of the form $(a, 0)$ on a line is called an x-*intercept *and a point of the form $(0, b)$ is called a y-*intercept*.*

Every line will have at least one intercept, but most have two. There are three special cases in which the line has only one intercept: if $A = 0$, $B = 0$, or $C = 0$. We will return to these special cases in a little bit.

Assume the line $Ax + By = C$ has both an x- and $y-$ intercept, we find them as follows:

- To find the x-intercept, we set $y = 0$ in the equation of the line and solve for x. Symbolically, this means that

$$x = \frac{C}{A}.$$

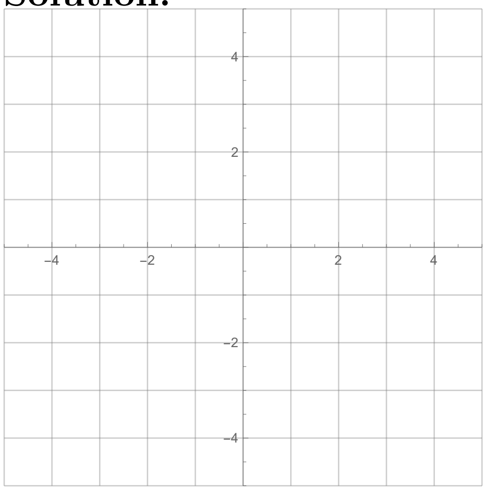
- To find the y-intercept, we set $x = 0$ in the equation of the line and solve for y. Symbolically, this means that

$$y = \frac{C}{B}.$$

To graph a line using intercepts, we plot the two intercepts in the xy-plane, and draw a line through the points:

Example 1. *Graph the line* $4x - 3y = 12$ *using intercepts.*

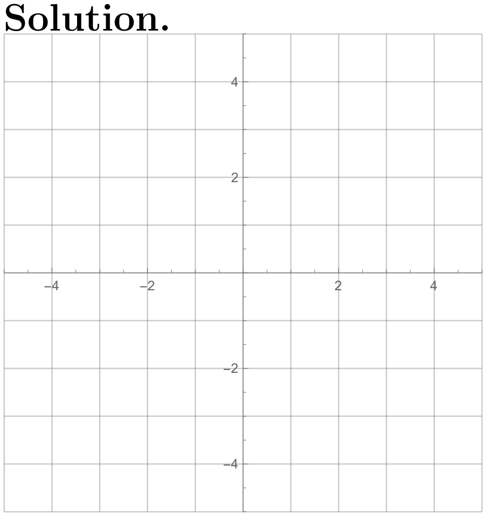
Solution.



Now, let's talk about one of those special cases, when $C = 0$ in $Ax + By = C$. We will assume that $A, B \neq 0$ here. If $C = 0$, you'll find that solving for the x-intercept as above gives $(0, 0)$ and solving for the y-intercept also gives $(0, 0)$. This means that both the x- and y- intercepts are at the origin. So, to graph the line $Ax + By = 0$, we need to come up with another point. You can really just pick any number other than 0 for x or y, then solve for the opposite. One easy thing that always works is to use one of the points $(B, -A)$ or $(-B, A)$, both are points on the line (check this!). That is, you just take the coefficients of x and y, flip their order, and put a minus sign in front of one of them. As a matter of fact, you could just use the two points $(B, -A)$ and $(-B, A)$ to graph the line. There's many other points you could use (which might be simpler than the previous two), but these always work.

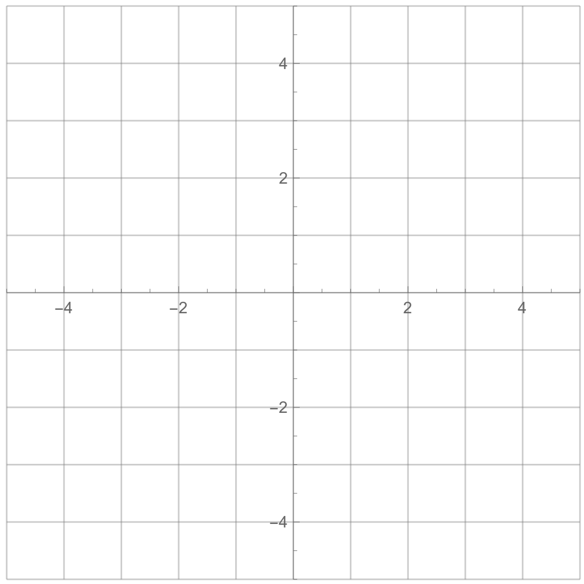
Example 2. *Graph the line* $2x + 3y = 0$.

Solution.

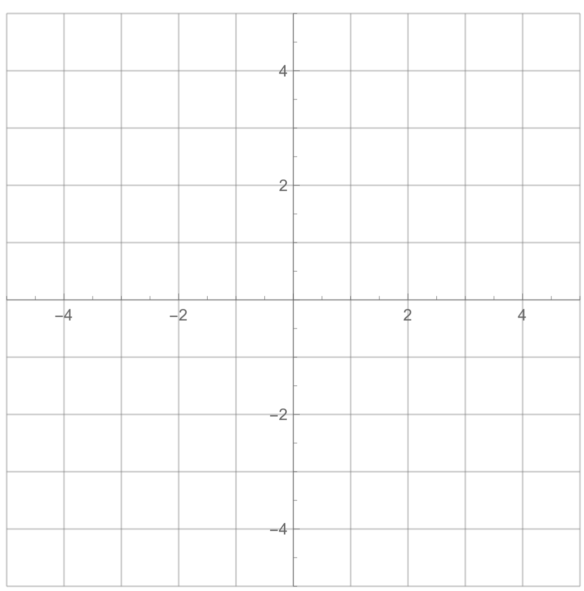


Horizontal and Vertical Lines. The cases when $A = 0$ or $B = 0$ in $Ax + By = C$ correspond to horizontal and vertical lines, respectively.

- If $A = 0$, we end up with the line $y = \dfrac{C}{B}$, which is a horizontal line where every y-value is $\dfrac{C}{B}$. A special one of these is when C is also zero so we get the equation $y = 0$. The graph of this line is the x-axis. Here are the graphs of $y = 2$ (red) and $y = -3$ (green).



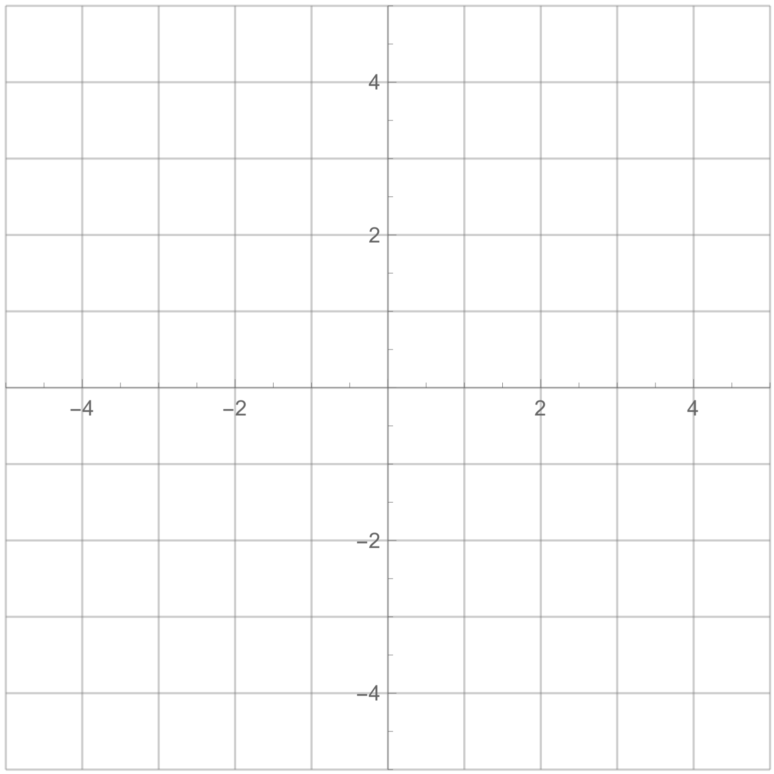
- If $B = 0$, we end up with the line $x = \dfrac{C}{A}$, which is a vertical line where every x-value is $\dfrac{C}{A}$. A special one of these is when C is also zero so we get the equation $x = 0$. The graph of this line is the y-axis. Here are the graphs of $x = 2$ (red) and $x = -3$ (green).



Example 3. *Graph the following lines:*

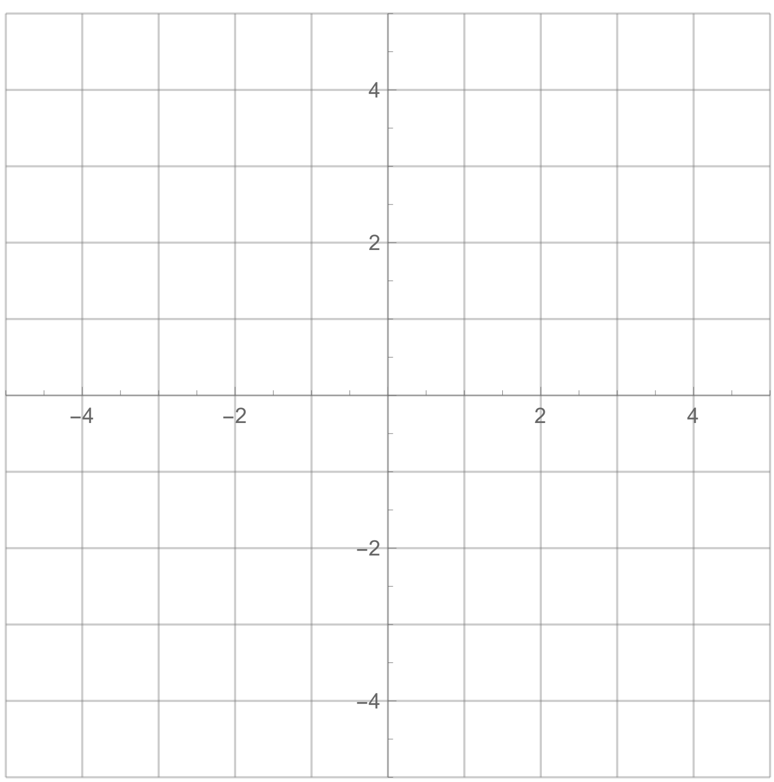
(a)

$$2x - y = 3$$



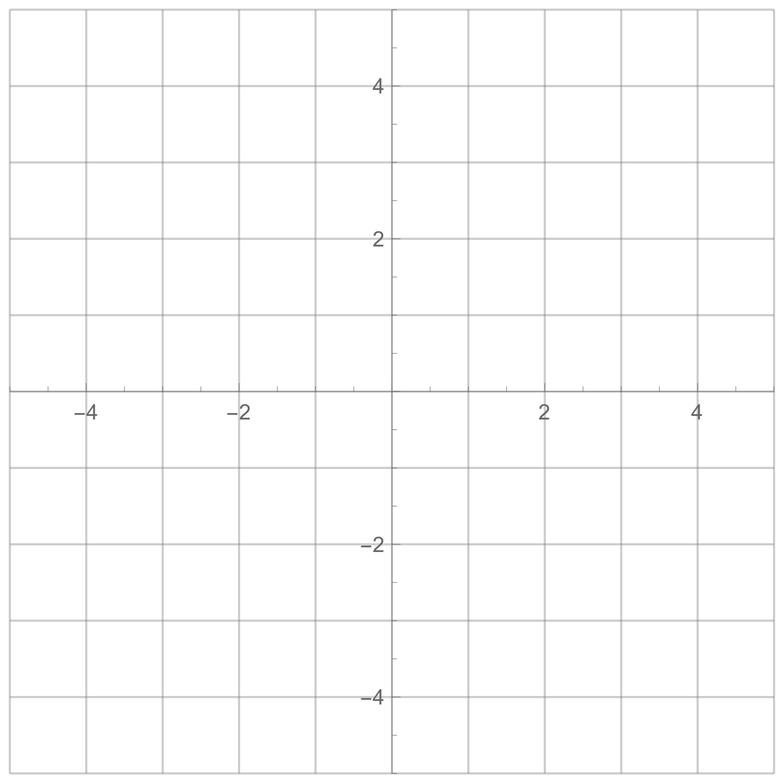
(b)

$$2x + 4y = 8$$



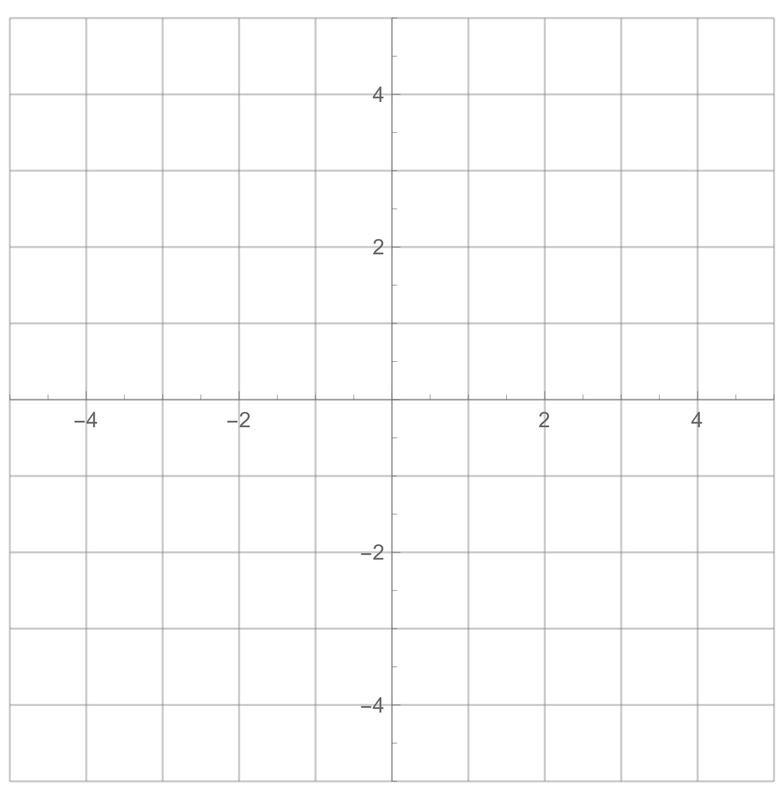
(c)

$$3x - 2y = 0$$



(d)

$$6x = 18$$



Section 4.1 - Systems of Linear Equations in Two Variables

Suppose we go to a movie theater and there are two packages for discounted tickets:
Package 1: 2 adult tickets and 1 child ticket for \$32
Package 2: 1 adult ticket and 3 child tickets for \$36

Based off of this information, can we figure how much the adult and child ticket discount prices are?

We can! To do this let A stand for the price of the adult ticket and let C stand for the price of the child ticket, then we get the following two equations from the two packages:

This is a system of two linear equations in two variables. To find the answer, we need to find a pair of numbers (A, C) which satisfy *both* equations simultaneously.

Definition 3 (System of Two Linear Equations in Two Variables)**.** *Given the* linear system

where a, b, c, d, h, and k are real constants, a pair of numbers $x = x_0$ *and* $y = y_0$ *(often written as an ordered pair* (x_0, y_0)*) is a* solution *of this system if each equation is satisfied by the pair. The set of all such ordered pairs is called the* solution set *for the system. To* solve *a system is to find its solution set.*

There are a few ways we can go about solving this: *graphically*, using *substitution*, and *elimination by addition*.

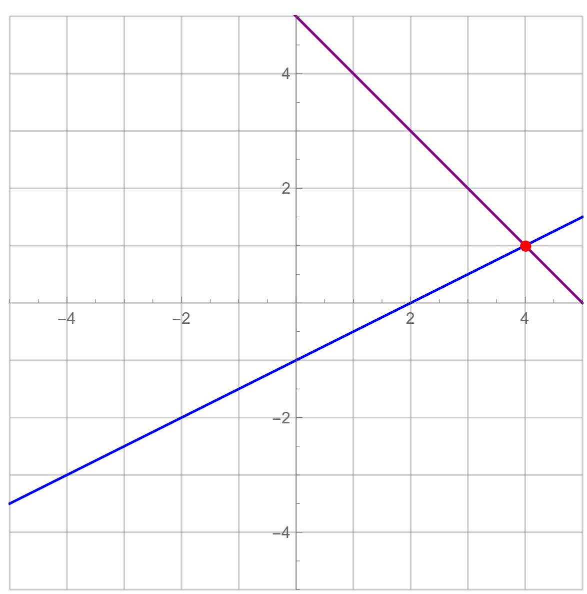
Solving by Graphing. To solve this problem by graphing, we simply graph the two equations in the system, then find the intersection. Since we're relying on a graph to find this point, we need to check our solution in the equations of the system.

There are actually 3 types of solutions to a system of linear equations

(1) Consider the system

$$\begin{array}{rcrcl} x & - & 2y & = & 2 \\ x & + & y & = & 5 \end{array}$$

If we graph the lines, we get

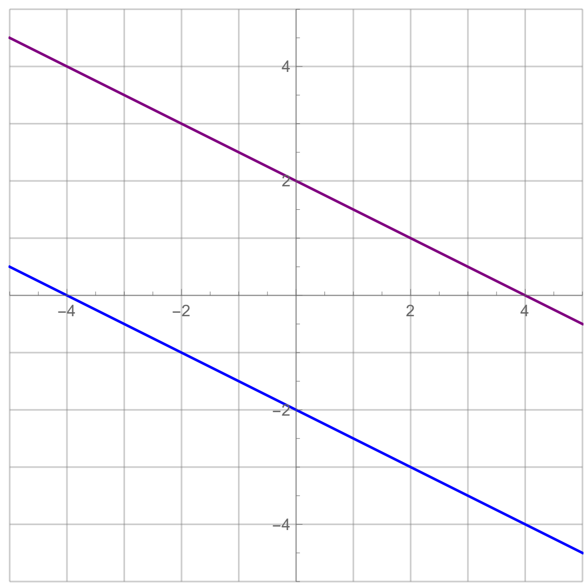


In this case, like before, we see only the *one solution* at $(4, 1)$. (You should check this in the system!)

(2) Consider the system

$$\begin{array}{rcrcl} x & + & 2y & = & 4 \\ 2x & + & 4y & = & 8 \end{array}$$

If we graph the lines, we get

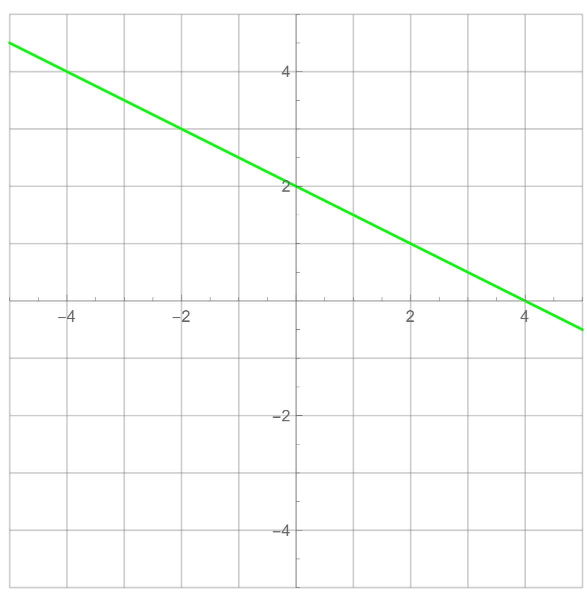


In this case, the lines are parallel and so they never intersect. In this case, there is *no solution*.

(3) Consider the system

$$\begin{array}{rcrcl} 2x & + & 4y & = & 8 \\ x & + & 2y & = & 4 \end{array}$$

If we graph the lines, we get



Here, both of the lines are exactly the same. In this case, there is an infinite number of solutions.

Definition 4. *A system of linear equations is called* consistent *if it has one or more solutions and* inconsistent *if it has no solutions. Further, a consistent system is called* independent *if it has exactly one solution (called the* unique solution*) and is called* dependent *if it has more than one solution. Two systems of equations are called* equivalent *if they have the same solution set.*

Theorem 1. *The linear system*

$$\begin{array}{rcrcl} ax & + & by & = & h \\ cx & + & dy & = & k \end{array}$$

must have

(1) *Exactly one solution (consistent and independent).*

(2) *No solution (inconsistent).*

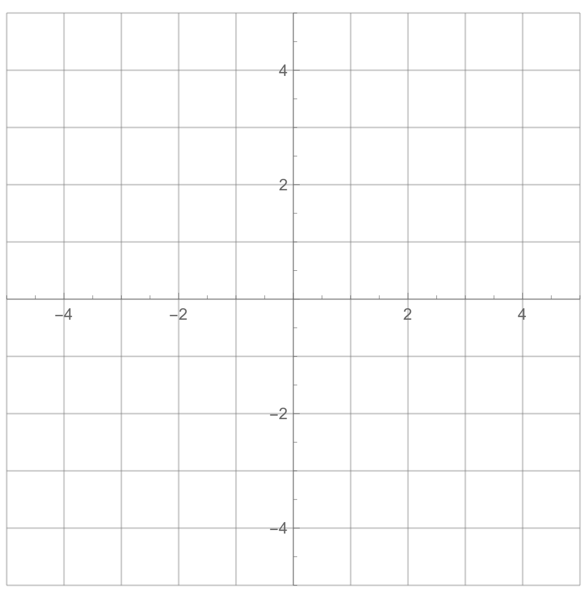
(3) *Infinitely many solutions (consistent and dependent).*

There are no other possibilities.

Example 4. *Solve the following systems of equations using the graphing method. Determine whether there is one solution, no solutions, or infinitely many solutions. If there is one solution, give the solution.*

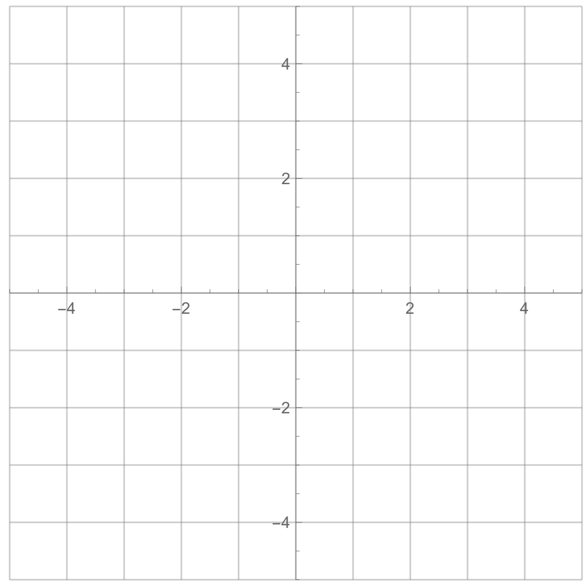
(a)

$$\begin{array}{rcrcl} x & + & y & = & 4 \\ 2x & - & y & = & 5 \end{array}$$



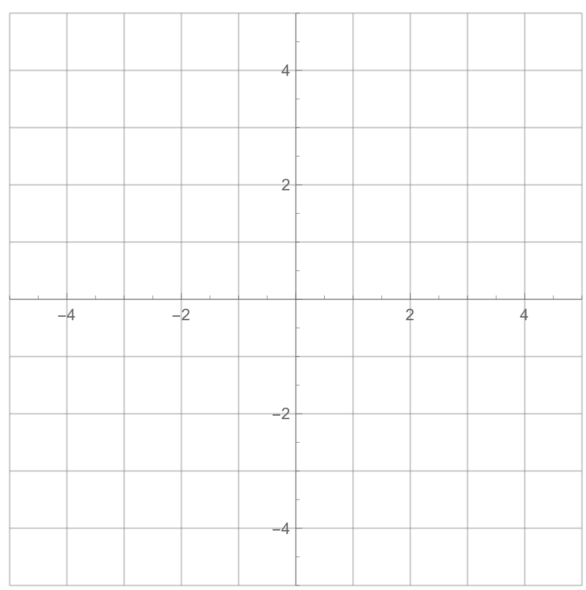
(b)

$$\begin{aligned} 6x - 3y &= 9 \\ 2x - y &= 3 \end{aligned}$$



(c)

$$\begin{aligned} 2x - y &= 4 \\ 6x - 3y &= -18 \end{aligned}$$



Solving by Substitution. When solving a system by substitution, we solve for one of the variables in one of the equations, then plug that variable into the other equation.

Example 5. *Solve the following system using substitution*

$$\begin{array}{rcrcl} 2x & - & y & = & 3 \\ x & + & 2y & = & 14 \end{array}$$

Example 6. *Solve the following system using substitution*

$$\begin{array}{rcrcl} 3x & + & 2y & = & -2 \\ 2x & - & y & = & -6 \end{array}$$

Solving Using Elimination. We now turn to a method that, unlike graphing and substitution, is generalizable to systems with more than two variables easily. There are a set of rules to follow when doing this

Theorem 2. *A system of linear equations is transformed into an equivalent system if*

(1) *two equations are interchanged*

(2) *an equation is multiplied by a nonzero constant*

(3) *a constant multiple of one equation is added to another equation.*

Example 7. *Solve the following system using elimination*

$$\begin{array}{rcrcr} 3x & - & 2y & = & 8 \\ 2x & + & 5y & = & -1 \end{array}$$

Example 8. *Solve the system using elimination*

$$\begin{array}{rcrcr} 5x & - & 2y & = & 12 \\ 2x & + & 3y & = & 1 \end{array}$$